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Reliability Based Optimum Design of a Gear Box

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ABSTRACT
The gear box represents an important mechanical sub system. In machine tools, the propose of a gear box is to provide a series of useful output speeds so that the machining operation can be carried out at its most optimum operating conditions high spindle speeds with low feed rate for roughing operations. An important aspect in the design of machine tool transmission is to keep the cost and volume of the gear box to a minimum.

The probabilistic approach to design has been considered to be more rational compared to the conventional design approach based on the factor of safety. The existence of uncertainties in either engineering simulations or manufacturing processes calls for a reliability-based design optimization (RBDO) model for robust and cost-effective designs.

In the present work a three shaft four speed gear box is designed using reliability principles. For the specified reliability of the system (Gear box), component reliability (Gear pair) is calculated by considering the system as a series system. Design is considered to be safe and adequate if the probability of failure of gear box is less than or equal to a specified quantity in each of the two failure modes. A FORTRAN program has been developed to calculate the mean values of face widths of gears for the minimum mass of gear box. By changing the probability of failure of system variations in the face widths are studied. The reliability based optimum design results are compared with those obtained by deterministic optimum design. The minimum mass of the gear box is increase as the specified values of the reliability is increased.
KEYWORDS: Reliability based optimum Design, four speed gear box.

I. INTRODUCTION

Optimization plays an important role in system design. In the design of any component or system, we will be interested either in maximizing the reliability subject to a constraint on the cost or in minimizing the cost with a restriction on the reliability of the system. In structural and mechanical design problems, the strength-based reliability becomes important. The standard optimization problem is stated first and then reliability based optimum design problems are formulated as optimization problems. Various techniques are available for the solution of optimum design problems.

The traditional design of gear boxes has been addressed extensively in the literature. Several aspects of gear optimization have also been considered. These include optimization of teeth form. Number of teeth, output speeds, gear train accelerations, end duty requirements.

The existence of uncertainties in either engineering simulations or manufacturing processes calls for a reliability-based design optimization (RBDO) model for robust and cost-effective designs. In the RBDO model for robust system parameter design, the mean values of the random system parameters are usually used as design variables, and the cost is optimized subject to prescribed probabilistic constraints by solving a mathematical nonlinear programming problem. Therefore, the solution from RBDO provides not only an improved design but also a higher level of confidence in the design.

Some work has been done in the area of probability, based mechanical design the probabilistic design of ellipsoidal and presser vessels was consider by Smith(2). The implementation of mechanical design to a reliability specification has been discussed by Mischke (12). In engineering design, the traditional deterministic design optimization model (Arora,(11) 1989) has been successfully applied to systematically reduce the cost and improve quality. Madhusekhar.D & Madhava Reddy.K(5) discussed the reliability based design of four speed and six speed gear boxes for the specified reliability and also studied the variations in face width of gears with the variation in design parameters.

II. PROBLEM FORMULATION

The problem of optimum design of the gear box is formulated according to deterministic and probabilistic approach.

2.1 Deterministic optimum design

The face widths of the gear box are treat as design variables and the optimum design problem is affirmed is non linear programming problem as follows:

Find $\mathbf{X}= \{x_1, x_2, x_3, \ldots x_8\}^T$

This minimizes the mass of the gear box

$$f(x) = \sum_{i=1}^{k} (\rho\, \pi D_i^2/4)\, x_I$$

Subjected to

$$s_{bij} - S_b \leq 0;\ i=1,2,3,\ldots\ldots,k,$$
$$j=1,2,3,\ldots\ldots.l,$$
$$s_{wij} - S_w \leq 0;\ i=1,2,3,\ldots\ldots,k,$$
$$j=1,2,3,\ldots\ldots.l,$$
$$x_i^{(l)} = x_i;\ i=1,2,3,\ldots\ldots.n$$

Where x_i = face width of ith gear

$x_i^{(l)}$ = lower bound on the face width of ith gear

The stresses induced s_{bij} and s_{wij} can be expressed as

$$S_{bij} = \beta_i M_{tij} / A_i x_i$$
$$s_{wij} = \gamma_i / A_i (M_{tij}/x_i)^{½}$$
$$M_t = 72735 * p / n_{wi}$$
$$\beta_i = [k_c k_d (r_i+1)]/[\ r_i m y_i \cos\alpha]$$
$$y_i = 0.52(1+20/T_{wi})$$
$$\gamma_i = 0.59\{(r_i+1)/r_i\} * [(ri+1) E k_c k_d / \sin 2\alpha]^{½}$$

2.2 Reliability based optimum design

The mean values of the face widths are treated as design variables and a linear combination of the expected value and standard deviations of the mass the mass of the gear box is considered for minimization. The design problem can be expressed as follows:

Find $\mathbf{X}= \{x_1, x_2, x_3, \ldots x_n\}^T$

Which minimizes

$$f(X) = c_1 \bar{f}(x) + c_2\, \sigma_f(x)$$
$$f(x) = \sum_{i=1}^{8} \rho \frac{\pi}{4} D_i^2\, x_i$$

$$f(x) = \rho\frac{\pi}{4} D_1^2 x_1 + \rho\frac{\pi}{4} D_2^2 x_2 + \rho\frac{\pi}{4} D_3^2 x_3 + \rho\frac{\pi}{4} D_4^2 x_4 + \rho\frac{\pi}{4} D_5^2 x_5 + \rho\frac{\pi}{4} D_6^2 x_6 + \rho\frac{\pi}{4} D_7^2 x_7 + \rho\frac{\pi}{4} D_8^2 x_8$$

where

$$D_1 = \frac{mT1}{2},\quad D_2 = \frac{mT2}{2},\quad D_3 = \frac{mT3}{2},\quad D_4 = \frac{mT4}{2},\quad D_5 = \frac{mT5}{2},\quad D_6 = \frac{mT6}{2},\quad D_7 = \frac{mT7}{2},\quad D_8 = \frac{mT8}{2},$$

$$\sigma_f(x) = \{\sum_{i=1}^{8}(\frac{\pi}{4} D_i^2\, x_i)^2\, \sigma_\rho^2 + \sum_{i=1}^{8}(\frac{\pi}{4}\rho D_i\, x_i)^2\, \sigma_{Di}^2 + \sum_{i=1}^{8}(\frac{\pi}{4}\rho D_i^2)^2\, \sigma_{xi}^2\}^{½}$$

Assume the co-efficient variation of diameters face width is 0.1 and co-efficient variation of density is 0.1, substituting the coefficient variations in objective function.

Finally equation is

$$f(X) = C_1[\rho\frac{\pi}{4}(D_1^2 x_1 + D_2^2 x_2 + D_3^2 x_3 + D_4^2 x_4 + D_5^2 x_5 + D_6^2 x_6 + D_7^4 x_7 + D_8^4 x_8] + C_2$$
$$[0.06(\rho\frac{\pi}{4})^2(D_1^4 x_1^2 + D_2^4 x_2^2 + D_3^4 x_3^2 + D_4^4 x_4^2 + D_5^4 x_5^2 + D_6^4 x_6^2 + D_7^4 x_7^2 + D_8^4 x_8^2)]$$

Let

x_1, x_3, x_5, x_7 are gear pairs then $x_1=x_2$, $x_3=x_4$, $x_5=x_6$, $x_7=x_8$

$$f(X) = C_1[\rho\frac{\pi}{4}\{(D_1^2 + D_2^2)\, x_1 + (D_3^2 + D_4^2)\, x_3 + (D_5^2 + D_6^2)\, x_5 + (D_7^2 + D_8^2)\, x_7\}] +$$
$$C_2[0.06(\rho\frac{\pi}{4})^2\{(D_1^4 + D_2^4)\, x_1^2 + (D_3^4 + D_4^4)\, x_3^2 + (D_5^4 + D_6^4)\, x_5^2 + (D_7^4 + D_8^4)\, x_7^2\}]$$

Subjected to

$$R_{b1} \geq R_0,\qquad R_{w1} \geq R_0,\qquad x_1 \geq 1$$
$$R_{b3} \geq R_0,\qquad R_{w3} \geq R_0,\qquad x_3 \geq 1,$$
$$R_{b5} \geq R_0,\qquad R_{w5} \geq R_0,\qquad x_5 \geq 1,$$
$$R_{b7} \geq R_0,\qquad R_{w7} \geq R_0,\qquad x_7 \geq 1$$

where C1 &C2 are constants denoting the relative importance's of the mean value and standard deviation of the mass during minimization, R_{bj} (R_{wj}) is the reliability of the gear box in jth speed in bending(surface wear) failure mode and R_o is the specified reliability. In the reliability based design optimization uses the reliability evaluated in the traditional reliability analysis to prescribe the probabilistic constraints. . By using a weakest link hypothesis, the reliability R_{bi} and R_{wj} can be expressed as

$$R_{bi} = \prod_{i=1}^{k} R_{bij}$$
$$R_{wj} = \prod_{j=1}^{k} R_{wij}$$

Where R_{bi} (R_{wj}) indicates the reliability of ith gear in jth speed in bending (surface wear) failure mode.

If the resistence (S) and the load acting (s) on a typical link of a weakest link chain are assumed to be independent normally distributed random variable, the reliability of the link (R_0) can determined as follows. A new random variable u is defined as

$$\mathbf{u = S - s}$$

Which implies that u is also normally distributed. If z denotes the standard normal variate

$$\mathbf{z = u - \bar{u}/\sigma_u}$$

The normally relation becomes

$$\int_{\infty}^{\infty} \mathbf{1/\sqrt{(2\pi)} * e^{(-t*t/2)dt=1}}$$

And the reliability of the link (R_0) can be expressed as (10)

$$\mathbf{R_0 = \int_{\infty}^{\infty}(u/\sigma u)(-Z^2/2)dZ = \int_{z1}^{\infty} 1/\sqrt{(2\pi)} * e^{(-t*t/2)dt}}$$

Where the lower limit of integration can be expressed in terms of the expressed values(S & s) and the standard deviations (σ_S , σ_s) of the random variables S and s as

$$\mathbf{z_1 = -\bar{u}/\sigma_u = -(\bar{S}-s)/\sqrt{(\sigma^2_S + \sigma^2_s)}}$$

If the reliability of the pair is known, the normal probability tables provide the value of the lower limit of integration, *z1*. In view of the relations and the fact that k/2 gear pairs are in mesh while delivering any of the output speeds, the constraints can be written as

$$\mathbf{(z_{1b})_{ij} \leq z_{10}}$$
$$\mathbf{(z_{1w})_{ij} \leq z_{10}}$$
$$\mathbf{(R_0) = \int_{\infty}^{\infty} exp(-z^2/2)dz}$$
$$\mathbf{(z_{1b})_{ij} = -(\bar{S}_b - \bar{s}_{bij})/\sqrt{(\sigma^2_{Sb} + \sigma^2_{sbij})}}$$
$$\mathbf{(z_{1w})_{ij} = -(\bar{S}_w - \bar{s}_{wij})/\sqrt{(\sigma^2_{Sw} + \sigma^2_{swij})}}$$

The mean values and standard deviations of *f*, s_{bij} . and s_{wij} are given in appendix. A and the value of z_{10} can be obtained from standard normal tables

III. FOUR SPEED GEAR BOX:

3.1 kinematic arrangement of a gear box

It is assumed that kinematic arrangement of four speed gear box is known and its shown in fig 3.1

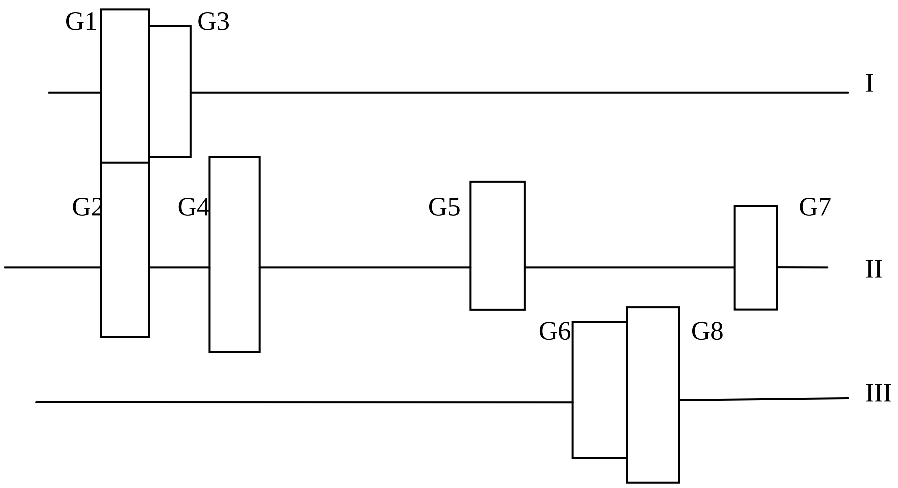

Fig 3.1 kinematic arrangement of 4 speed gear box

3.2 The following data is considered while designing a 4 speed gear box:

The values of power(P),bending strength(S_b),wear strength(S_w) were taken as 10 H.P,2500 Kg/cm^2 and 17500 Kg/cm^2 respectively and K_c =1.5,K_d=1.1, α=20^o,C_P = C_{nw}= C_{sw} =0.1, C_t=C_a=0.1,E=2.1x10^6 Kg/cm^2 .Speeds are 400rpm,560rpm,800rpm,1120rpm. Number of gear teeth of a gearbox G1=24, G2=24, G3=20, G4=28, G5=27, G6=27, G7=18, G8=36 ρ=7.75x10^{-3} Kg/cm^2, Torques's of 4 gear pairs is 17.9kg-m, 12.79kg-m, 8.95kg-m, 6.39kg-m, and assume the standard module m=4mm

The mean values and standard deviations of the power transmitted by the gear train and the material properties are assumed to be known as design data .These same values are used in the deterministic optimum design. All the random variables are assumed to be independent and normally distributed in the solution of reliability based optimization problems.

IV. OPTIMIZATION PROCEDURE

Both the deterministic and probabilistic optimization problems formulated in the previous section can be stated in the form of standard nonlinear programming problems. The Lagrange multiplier method is an analytical method that can be used to find the minimum of a multivariable function in the presence of a equality constraints. Thus the problem to be solved can be stated as

Find $\mathbf{X = \{x_1\ x_2\ x_3 \ldots x_n\}^T}$

Which minimizes

$$\mathbf{f(X)}$$

$$\mathbf{l_j(X) = 0;\quad j = 1, 2, 3, \ldots, p} \qquad \mathbf{(1)}$$

The solution procedure involves construction of a new function, called the Lagrange function, L(X,λ), as

$$\mathbf{L(X,\lambda)\ f(X) + \sum_{j=1}^{p} \lambda_j l_j(X)} \qquad \mathbf{(2)}$$

Where λ_j are unknown constants, called the Lagrange multipliers. Notices that there are as many Lagrange multipliers as there are constraints. It has been proved that the solution of the problem stated in eq.(1) can be obtained by find the unconstrained minimum of the function L(X,λ). The necessary conditions for the minimum of L(X,λ) are

$$\mathbf{\partial L/\partial x_i(X,\lambda) = 0\ ;\quad i=1, 2, 3, \ldots, n}$$

$$\mathbf{\partial L/\partial x_j(X,\lambda) = 0\ ;\quad j=1, 2, 3, \ldots, p} \qquad \mathbf{(3)}$$

The number of unknowns is n + p (n design variables x_i and p Lagrange multipliers λ_j) and the number of available equations, eq (3), is also n + p. thus the solution of the (n + p) simultaneous nonlinear equations (3) gives X* and λ*. Although this method appears to be simple and straightforward, the solution of the (n+p) simultaneous nonlinear equations, eq (3). may be tedious. Further, if the functions f and l_j are not available in explicit form in terms of x_i, it will be extremely difficult to solve eq.(3). Hence, a suitable numerical method of optimization.

V. RESULTS

The values of the face widths of the gear pairs are calculated using a FORTRAN program. Considering the coefficient of variation of all random variables is 0.1.

1. For the deterministic design and the power is kept as constant (P = 10 HP) and the factor of safety is varied from 1.5 to 4 and the face width obtained for these values are tabulated and also represented by graph. the factor of safety is applied to the material strength and the strengths are taken as S_b=2500kg/cm^2,S_w=17500kg/cm^2.
2. The coefficient of variation of speed and the power are kept as constant (C_{nw}=0.1, P = 10 HP, and assume c1=1, c2=100) and the probability of failure is varied from 1x10^{-1} to 1x10^{-6} and the face width obtained for these values are tabulated and also represented by graph.
3. The reliability based optimum face width and minimum mass results are obtained and the values are tabulated with three different C1 and C2 values.(C1=1,C2=100 & C1=0,C2=100 & C1=1,C2=0)
4. The face width and mass obtained with deterministic optimum design are also tabulated.

5.1 DETERMINISTIC DESIGN FOR 4 SPEED GEAR BOX

The face widths of gear pairs in a 4 speed gear box obtained by varying FS from 1.5 to 4 are shown in Table 5.1.

Input given: Pressure angle=20 degrees; Power=10HP; Z=1.638

TABLE: 5.1 Face width of gear pairs with variation of Factor of Safety:

Factor of Safety	GEAR PAIR				MASS (kg)
	(G1,G2) (cm)	(G3,G4) (cm)	(G5,G6) (cm)	(G7,G8) (cm)	
1.5	0.7771	0.9590	0.8596	1.4507	1.3717
2	1.3618	1.704	1.5283	2.5776	2.4323
2.5	2.1588	2.6640	2.3880	4.0297	3.8105
3	3.1087	3.8362	3.4387	5.8010	5.4865
3.5	4.2312	5.2215	4..6805	7.8958	7..4677
4	5.5265	6.8199	6.1133	10.3162	9.7553

5.2 Reliability based optimum design

The face widths of gear pairs in a 4 speed gear box obtained by varying the probability of failure from $1x10^{-1}$to$1x10^{-6}$ are tabulated in Table 5.2.

Input given: Pressure angle=20 degrees; power=10HP; C_{nw}=0.1 ; C_1=1; C_2=100

TABLE: 5.2 Face width of gear pairs with variation of probability of failure:

GEAR PAIR	PROBABILITY OF FAILURE					
	$1X10^{-1}$	**$1X10^{-2}$**	**$1X10^{-3}$**	**$1X10^{-4}$**	**$1X10^{-5}$**	**$1X10^{-6}$**
(G1,G2) (cm)	0.5754	0.7738	0.9774	1.2025	1.4607	1.7543
(G3,G4) (cm)	0.7101	0.9550	1.206	1.4841	1.8022	2.1641
(G5,G6) (cm)	0.6364	0.8559	1.0811	1.3301	1.6155	1.9401
(G7,G8) (cm)	1.0736	1.4442	1.8241	2.2438	2.7226	3.2686
F(x) (kg)	1.0155	1.3658	1.7251	2.1223	2.5766	3.0938
MASS (kg)	1.0154	1.3657	1.7250	2.1222	2.5765	3.0938

The face widths of gear pairs in a 4 speed gear box obtained by varying the probability of failure from $1x10^{-1}$to$1x10^{-6}$ are tabulated in Table 5.3.

Input given: Pressure angle=20 degrees; power=10HP; C_{nw}=0.1 ; C_1=1; C_2=0

TABLE: 5.3 Face widths of gear pairs with variation of probability of failure:

GEAR PAIR	PROBABILITY OF FAILURE					
	$1X10^{-1}$	**$1X10^{-2}$**	**$1X10^{-3}$**	**$1X10^{-4}$**	**$1X10^{-5}$**	**$1X10^{-6}$**
(G1,G2) (cm)	0.5754	0.7738	0.9774	1.2025	1.4607	1.7543
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(G7,G8) (cm)	1.0736	1.4442	1.8241	2.2438	2.7226	3.2686
. F(x) (kg)	1.0155	1.3658	1.7251	2.1223	2.5766	3.0938
.MASS (kg)	1.0154	1.3657	1.7250	2.1222	2.5765	3.0938

The face widths of gear pairs in a 4 speed gear box obtained by varying the probability of failure from $1x10^{-1}$to$1x10^{-6}$ are tabulated in Table 5.4.
Input given: Pressure angle=20 degrees; power=10HP; C_{nw}=0.1 ; C_1=0 ;C_2=100

TABLE: 5.4 Face widths of gear pairs with variation of probability of failure

GEAR PAIR	**PROBABILITY OF FAILURE**					
	$1X10^{-1}$	**$1X10^{-2}$**	**$1X10^{-3}$**	**$1X10^{-4}$**	**$1X10^{-5}$**	**$1X10^{-6}$**
(G1,G2) (cm)	0.5754	0.7738	0.9774	1.2025	1.4607	1.7543
(G3,G4) (cm)	0.7101	0.9550	1.206	1.4841	1.8022	2.1641
(G5,G6) (cm)	0.6364	0.8559	1.0811	1.3301	1.6155	1.9401
(G7,G8) (cm)	1.0736	1.4442	1.8241	2.2438	2.7226	3.2686
F(x) (kg)	1.2535	1.6860	2.1296	2.6198	3.1802	3.8184
MASS (kg)	1.0155	1.3658	1.7251	2.1223	2.5766	3.0938

5.3 Deterministic optimum design:

Deterministic optimum design is carried out by considering the values of design variables and assuming on variation.

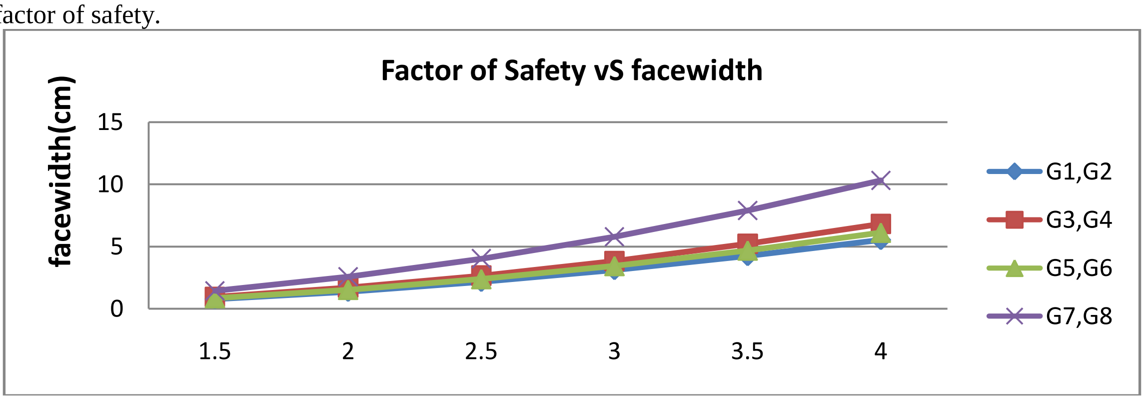
The face widths of a gear pair in a 4 speed gear box obtained are tabulated in the table 5.5

TABLE: 5.5 Face widths of gear pairs

GEAR PAIR	FACE WIDTH(cm)	MASS (kg)
(G1,G2)	0.3452	0.6095
(G3,G4)	0.4261	
(G5,G6)	0.3819	
(G7,G8)	0.6445	

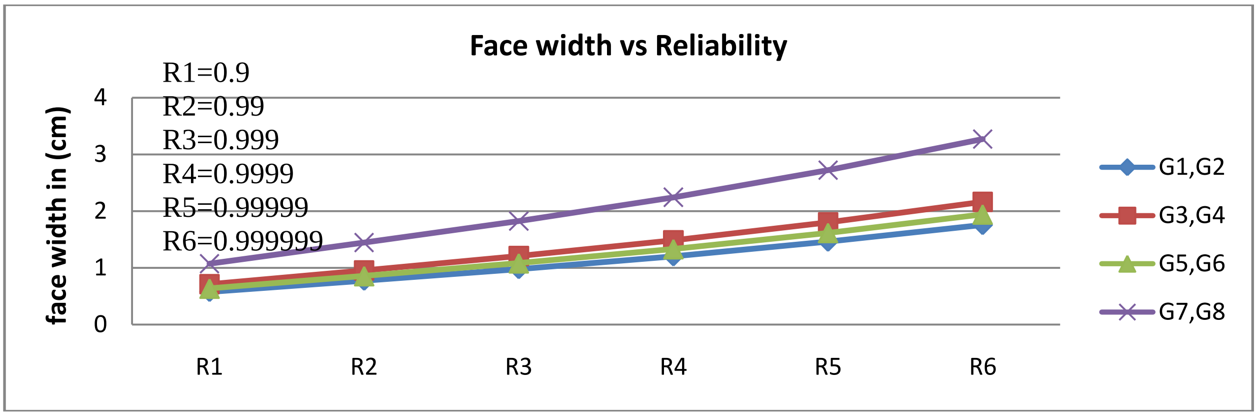
5.4 Graphs

The fig 5.1 shows the effect of factor of safety on face width of gear pairs. Face width is taken on Y-axis and factor of safety is taken on X-axis. It is clear that the face widths of gear pairs increase with increase in factor of safety.



5.1 Effect of factor of safety on face width of gear pairs

fig 5.2 shows the effect of reliability on face width of gear pairs. Face width is taken on Y-axis and Reliability is taken on X-axis. It is evident that the face widths of gear pairs increase with increase in reliability.



5.2 Effect of reliability on face width of gear pairs

6.3. Conclusion

1. The present result indicates that the minimum mass of the gear box will be higher in the case of probabilistic design compared to that of the deterministic design.
2. The reliability based optimum design results obtained with three different C1 and C2 values. It can be observed that all combinations of C1 and C2 yield essentially the same optimum face width and mass.
3. The probabilistic optimum design procedure is quite general and can be applied to any machine element.
4. It is evident from the above results that as the reliability increases face width of the gear pair and mass increases.
5. It is evident from the above results that as the factor of safety increases face width of the gear pair and mass increases.
6. Procedure is generally we can be applied to any mechanical system.

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NOTATIONS

P=Power in H.P
n_w= wheel speed
F.S=factor of safety
M_{tij}=Torque acting on the gear teeth
s_{bij}=Induced bending stress ith gear in jth pair
A_i=center distance between the gears
K_c=stress concentration factor
K_d=dynamic load factor
r_i=transmission ratio of ith gear pair
T_{wi}=no. of teeth on the wheel
T_{pi}=no. of teeth on the pinion
α=pressure angle
Y_i=lewis factor for ith gear pair
s_{wij}=induced wear stress in ith gear in jth pair
E=young's modulus of the material of the gear
f(X) = objective function
n = number of design variable
Z=standard normal variable

x_i=face width of ith gear pair(cm)
ρ =mass density of the material (kg/cm^2)
X=random variable
$\overline{x}$=mean value of x
σ_x =standard deviation of x
C_x=coefficient of variation of x
Pf=probability of failure
R=reliability= (1-pf)
C_1, C_2 =constants
K= number of gears